A hybrid spin-electron system on one-dimensional tetrahedral chain, in which the localized Ising spin regularly alternates with the mobile electron delocalized over three lattice sites, is exactly solved using the generalized decoration-iteration transformation and the transfer-matrix technique. It is shown that the investigated system exhibits either the ferromagnetic or antiferromagnetic ground state, depending on whether the ferromagnetic or antiferromagnetic interaction between the Ising spins and mobile electrons is considered. Moreover, the enhanced magnetocaloric effect observed during the adiabatic demagnetization suggests a potential use of the studied spin-electron model for low-temperature magnetic refrigeration.